### 1.6.4 Numerical Differential Equations

\[\text{NDSolve}[\text{eqns, } y, \{x, \text{xmin, xmax}\}]\]

- solves numerically for the function \(y\), with the independent variable \(x\) in the range \(\text{xmin} \) to \(\text{xmax}\)

\[\text{NDSolve}[\text{eqns, } \{y_1, y_2, \ldots\}, \{x, \text{xmin, xmax}\}]\]

- solves a system of equations for the \(y_i\)

**Numerical solution of ordinary differential equations.**

This generates a numerical solution to the equation \(y'(x) = y(x)\) with \(0 < x < 2\).

The result is given in terms of an \texttt{InterpolatingFunction}.

Here is the value of \(y(1.5)\).

With an algebraic equation such as \(x^2 + 3x + 1 = 0\), each solution for \(x\) is simply a single number. For a differential equation, however, the solution is a function, rather than a single number. For example, in the equation \(y'(x) = y(x)\), you want to get an approximation to the function \(y(x)\) as the independent variable \(x\) varies over some range.

\texttt{Mathematica} represents numerical approximations to functions as \texttt{InterpolatingFunction} objects. These objects are functions which, when applied to a particular \(x\), return the approximate value of \(y(x)\) at that point. The \texttt{InterpolatingFunction} effectively stores a table of values for \(y(x_i)\), then interpolates this table to find an approximation to \(y(x)\) at the particular \(x\) you request.

\[\texttt{y[x] /. solution}\]

use the list of rules for the function \(y\) to get values for \(y[x]\)

\[\texttt{InterpolatingFunction[data}[x]\]

evaluate an interpolated function at the point \(x\)

\[\texttt{Plot[Evaluate[y[x] /. solution]], \{x, \text{xmin, xmax}\]}\]

plot the solution to a differential equation

Using results from \texttt{NDSolve}.

This solves a system of two coupled differential equations.

\[\text{In}[3]:= \text{NDSolve}[\{y'[x] == z[x], z'[x] == -y[x], y[0] == 1, \quad z[0] == 1, \quad \{y, z\}, \{x, 0, \Pi\}\}]\]

\[\text{Out}[3]= \{(y -> \texttt{InterpolatingFunction}[\{0., 3.14159\}, \{\}\}], \quad (z -> \texttt{InterpolatingFunction}[\{0., 3.14159\}, \{\}\]}\)
Here is the value of \( z[2] \) found from the solution.

Here is a plot of the solution for \( y[x] \) found on line 1. \texttt{Plot} is discussed in Section 1.9.1.

\begin{verbatim}
In[4]:= z[2] /. %
Out[4]= \{-0.416167\}

In[5]:= Plot[Evaluate[y[x] /. %], {x, 0, 2}]
\end{verbatim}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{plot.png}
\caption{Plot of the solution for \( y[x] \).}
\end{figure}